

Towards using data assimilation in macro-economic dynamical models

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Outline

- 1 The NEDyM model
- 2 Twin data assimilation experiments with the NEDyM model
- 3 Plans for NEDyM assimilation of real observations

Prognostic variables

- ▶ **NEDyM**: A macro-economic non-equilibrium **dynamical** model that has been shown to exhibit **endogenous business cycles** (see talk by P. Dumas, [Hallegatte et al., 2008; Groth et al., 2012]).
- ▶ The prognostic variables are those of the dynamical model (that are used to write down its dynamics).
- ▶ Seven prognostic variables (small dimensionality):
 - F : Stock of liquid assets
 - H : Goods inventory
 - K : Capital (infrastructure + money)
 - L : Employment rate
 - p : Price
 - w : Real wage
 - Γ_{inv} Investment ratio dynamics

Diagnostic equations

► Intermediary diagnostic variables:

$$M = C_0 - F \quad \text{Consumer stock of money}$$

$$C = (1 - \gamma_{\text{save}}) \frac{1}{p} \alpha_M M \quad \text{Consumer consumption}$$

$$I = \frac{1}{p} \Gamma_{\text{inv}} \alpha_F F \quad \text{Investment}$$

$$D = C + I \quad \text{Demand (sales)}$$

$$Y = AL^\lambda K^{1-\lambda} \quad \text{Production}$$

$$\Pi_n = pD - wL - \frac{1}{\tau_{\text{dep}}} pK \quad \text{Expected net profit}$$

Prognostic equations

$$\dot{H} = Y - D \quad \text{Goods inventory} \quad (1)$$

$$\dot{p} = -p\alpha_p \frac{H}{D} \quad \text{Price} \quad (2)$$

$$\dot{L} = -\frac{1}{\tau_{empl}}(L - L_e) \quad \text{Employment rate} \quad (3)$$

$$\dot{w} = \frac{w}{\tau_{wage}} \frac{L - L_{full}}{L_{full}} \quad \text{Wage} \quad (4)$$

$$\dot{K} = I - \frac{1}{\tau_{dep}}K \quad \text{Capital} \quad (5)$$

$$\dot{F} = pD - wL + \gamma_{save}\alpha_M M - (1 - \Gamma_{inv})\alpha_F F - pl \quad \text{Stock of Liquid assets} \quad (6)$$

$$\dot{\Gamma}_{inv} = \begin{cases} \alpha_{inv}(\gamma_{max} - \Gamma_{inv})(\Pi_n/(pK) - v) & \text{if } \Pi_n/(pK) > v \\ \alpha_{inv}(\gamma_{max} - \Gamma_{inv})(\Pi_n/(pK) - v) & \text{if } \Pi_n/(pK) \leq v \end{cases} \quad \text{Investment ratio} \quad (7)$$

Diagnostic variables

► Diagnostic variables: the ones that are accessible and possibly observed.
There must be an observation operator \mathbf{H} that links the prognostic variables \mathbf{x} to the diagnostic variables \mathbf{y} .

► The diagnostic variables (also defines \mathbf{H}):

$$Y = AL^\lambda K^{1-\lambda} \quad \text{Production, GDP } \lambda = 2/3 \quad (8)$$

$$I = \frac{1}{p} \Gamma_{\text{inv}} \alpha_F F \quad \text{Investment} \quad (9)$$

$$L/L_{\text{max}} \quad \text{Employment} \quad (10)$$

$$C = (1 - \gamma_{\text{save}}) \frac{1}{p} \alpha_M M \quad \text{Consumer consumption} \quad (11)$$

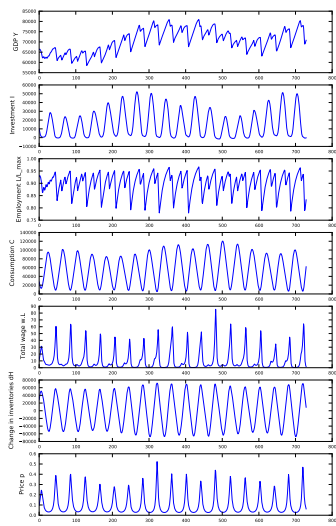
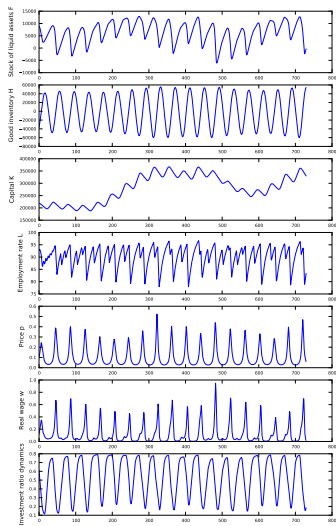
$$wL \quad \text{Total wage} \quad (12)$$

$$\dot{H} = Y - (C + I) \quad \text{Change in inventory} \quad (13)$$

$$p \quad \text{Price} \quad (14)$$

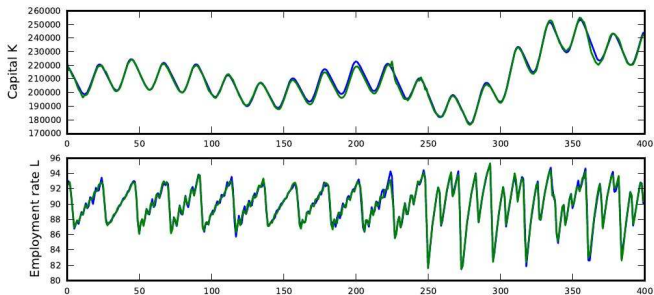
► A nonlinear observation operator!

Prognostic and diagnostic variables



Characteristics of NEDyM with a view to data assimilation

- ▶ Nonlinear evolution model and nonlinear observation operator, with thresholds, and non-integer power laws (non-differentiability).
- ▶ Dynamics: Equilibrium ($\alpha_{inv} < 1.39$), Limit cycles ($1.39 < \alpha_{inv} < 3.8$), chaotic behaviour ($3.8 < \alpha_{inv}$)



Characteristics of NEDyM with a view to data assimilation

- ν : Standard of profitability
 - γ_{save} : Saving ratio
 - τ_{dep} : Capital depreciation time
 - τ_{wage} : Wage characteristic time
 - τ_{empl} : Employment characteristic time
 - α_p : Price adjustment coefficient
 - α_F : Using rate of the producer liquid assets
 - α_{inv} : Produce investment ratio
 - C_0 : Total stock of money and liquid assets
 - A : Total productivity
 - L_{full} : Full employment
 - L_{max} : Total number of workers
 - L_e : Optimal labour demand
- ▶ 13 parameters within the model's equations! Need to be tuned or estimated. 7 parameters are determined by the calibration of the model equilibrium at $\alpha_{\text{inv}} = 0$ to the EU economy in 2001. The other 6 have been chosen in an *ad hoc* manner, so far.

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Far-reaching data assimilation objectives

▶ Stepping stones (**with synthetic data**):

- 1 Choice of the data assimilation method,
- 2 Consistency/adequacy tests,
- 3 Stability/performance of the filtering scheme,
- 4 Robustness of the model to perturbations.

▶ Far-reaching objectives (**with real data**):

- 1 Adjust some of the model's key parameters, relying on data assimilation and using real data,
- 2 Perform an economic forecast based on the model thus adjusted,
- 3 And perform a re-analysis of past economic history.

The data assimilation system

- ▶ The dynamical model and the observations:

$$\mathbf{x}_{k+1} = M_{k+1 \leftarrow k}(\mathbf{x}_k) + \eta_k \quad (15)$$

$$\mathbf{y}_k = H_k(\mathbf{x}_k) + \varepsilon_k \quad (16)$$

- ▶ Statistics of the errors in the context of a twin experiment
 - Perfect model assumption ($\eta_k = \mathbf{0}$), or small stochastic perturbations.
 - Fully observed system every quarter of year.
 - Normal error on each of the diagnostic variables, with a standard deviation equal to the climatological variability (20%).

Choosing the data assimilation method

► The most natural one: the extended Kalman filter (see [Ghil and Malanotte-Rizzoli, 1991] in geophysics).

① Initialisation: State of the system \mathbf{x}_0^f and error covariance matrix \mathbf{P}_0^f .

② for $t_k = 1, 2, \dots$

① Analysis

- Gain computation $\mathbf{K}_k = \mathbf{P}_k^f \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_k^f \mathbf{H}_k^T + \mathbf{R}_k)^{-1}$

- State estimation: $\mathbf{x}_k^a = \mathbf{x}_k^f + \mathbf{K}_k (\mathbf{y}_k - H_k[\mathbf{x}_k^f])$

- Compute analysis error covariance matrix: $\mathbf{P}_{k+1}^f = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_k^f$

② Forecast

- State forecast $\mathbf{x}_{k+1}^f = M_{k+1}[\mathbf{x}_k^a]$

- Error covariance matrix forecast: $\mathbf{P}_{k+1}^f = \mathbf{M}_{k+1} \mathbf{P}_k^a \mathbf{M}_{k+1}^T + \mathbf{Q}_k$

► It has drawbacks:

- Could be costly (about 14 propagations of the TL model in each cycle) for extensive tests.

- Propagation of the errors by the linear tangent model.

- The observation operator is linearised.

Alternative: the ensemble Kalman filter (one cycle)

1 Observation [Evensen, 1994]

- Observation sets $j = 1, \dots, N$: $\mathbf{z}_j = \mathbf{z} + \mathbf{u}_j$ $\sum_{j=1}^N \mathbf{u}_j = 0$
- Related error covariance matrix: $\mathbf{R} = \frac{1}{N-1} \sum_{j=1}^N \mathbf{u}_j \mathbf{u}_j^T$

2 Analysis

- Gain computation $\mathbf{K} = \mathbf{P}^f \mathbf{H}^T (\mathbf{H} \mathbf{P}^f \mathbf{H}^T + \mathbf{R})^{-1}$
- State estimation for $j = 1, \dots, N$: $\mathbf{x}_j^a = \mathbf{x}_j^f + \mathbf{K} (\mathbf{z}_j - H(\mathbf{x}_j^f))$, $\bar{\mathbf{x}}^a = \frac{1}{N} \sum_{j=1}^N \mathbf{x}_j^a$
- Error covariance matrix computation $\mathbf{P}^a = \frac{1}{N-1} \sum_{j=1}^N (\mathbf{x}_j^a - \bar{\mathbf{x}}^a) (\mathbf{x}_j^a - \bar{\mathbf{x}}^a)^T$

3 Forecast

- State forecast for $j = 1, \dots, N$ $\mathbf{x}_j^f = M(\mathbf{x}_j^a)$, $\bar{\mathbf{x}}^f = \frac{1}{N} \sum_{j=1}^N \mathbf{x}_j^f$
- Error covariance matrix estimation: $\mathbf{P}^f = \frac{1}{N-1} \sum_{j=1}^N (\mathbf{x}_j^f - \bar{\mathbf{x}}^f) (\mathbf{x}_j^f - \bar{\mathbf{x}}^f)^T$

Even better: the deterministic ensemble Kalman filter (one analysis)

The analysis does not require the perturbations of observations [Whitaker and Hamill, 2002]
 + ensemble transform form [Hunt et al., 2007]:

- 1 Requires: The forecast ensemble $\{\mathbf{x}_k\}_{k=1,\dots,N}$, the observations \mathbf{y} , and error covariance matrix \mathbf{R}
- 2 Compute the mean $\bar{\mathbf{x}}$ and the anomalies \mathbf{A} from $\{\mathbf{x}_k\}_{k=1,\dots,N}$.
- 3 Compute $\mathbf{Y} = \mathbf{H}\mathbf{A}$, $\delta = \mathbf{y} - \mathbf{H}\bar{\mathbf{x}}$
- 4 Compute $\mathbf{w}_a = (\mathbf{Y}^T\mathbf{R}^{-1}\mathbf{Y} + N - 1)^{-1}\mathbf{Y}^T\mathbf{R}^{-1}\delta$.
- 5 Compute $\mathbf{x}_a = \bar{\mathbf{x}} + \mathbf{A}\mathbf{w}_a$.
- 6 Compute $\mathbf{\Omega}_a = \{\mathbf{Y}^T\mathbf{R}^{-1}\mathbf{Y} + N - 1\}^{-1}$
- 7 Compute $\mathbf{W}^a = \{(N - 1)\mathbf{\Omega}_a\}^{1/2}\mathbf{U}$
- 8 Compute $\mathbf{x}_k^a = \mathbf{x}^a + \mathbf{A}\mathbf{W}_k^a$

Advantages and drawbacks

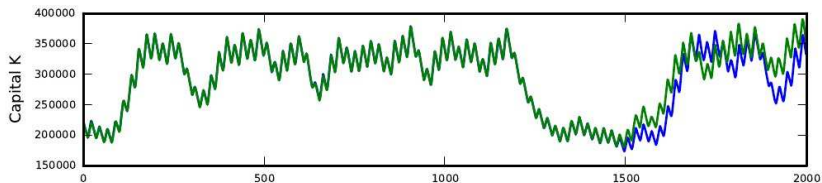
► Advantages

- 1 Cheap: $N = 10$ ensemble propagation by cycle.
- 2 This ensemble size is sufficient for NEDyM.
- 3 Propagation of the ensemble by the full model.
- 4 No adjoint of the observation operator is required (approximated by the ensemble).
- 5 Ensemble transform form handles very heterogeneous variables (ideal for NEDyM); better conditioning of the analysis.

► Drawbacks

- 1 Inflation may be required (especially in the chaotic regime)
- 2 Inefficient initialisation.

Inflation

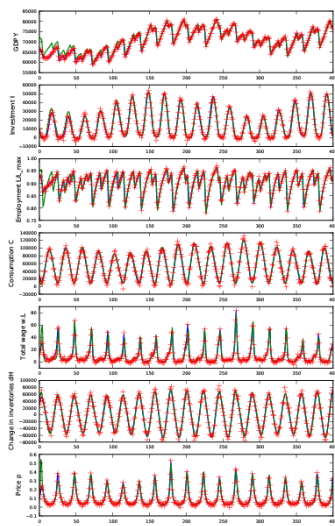
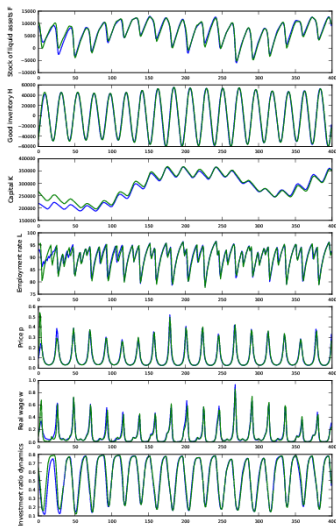


- ▶ A very small inflation needed in the chaotic regime ($\alpha_{\text{inv}} = 5$) for a 500 hundred years run: $\mathbf{x}_k \rightarrow \bar{\mathbf{x}} + \lambda(\mathbf{x}_k - \bar{\mathbf{x}})$ with $\lambda = 1.001$.
- ▶ Alternatively, one can use the finite-size EnKF (EnKF-N) which does not require inflation [Bocquet, 2011].
- ▶ Seems an academic problem, but inflation will again be needed in the presence of model error.
- ▶ This shows that the Gaussian filtering pdf is barely distorted by the nonlinear flow. This implies that the difficulty of data assimilation with NEDyM is not in its chaotic flow but rather in its nonlinear (even non-differentiable) operators.

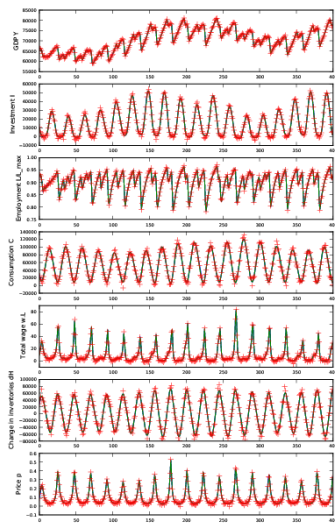
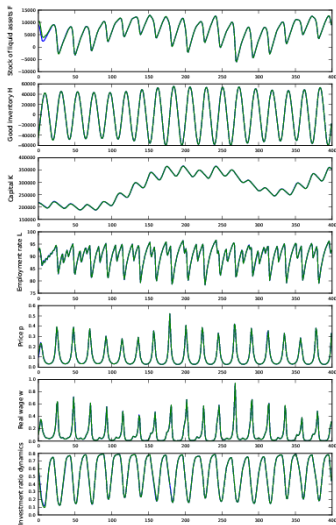
Initialisation (1/3)

- ▶ The EnKF is a reduced-rank method. It cannot span the whole phase space. As a consequence, if the initial condition of the DA method is far from the truth, the filter may diverge or take time to recover.
- ▶ An adaptive technique such as EnKF-N or EnKF-ML may be superior in that respect.
- ▶ An alternative is to use inflation (but how much?)

Initialisation: EnKF (2/3)



Initialisation: EnKF-N (3/3)



Model error (1/4)

- ▶ We assume the truth obeys to

$$\mathbf{x}_{k+1}^t = M_{k+1 \leftarrow k}(\mathbf{x}_k^t) + \eta_k, \quad (17)$$

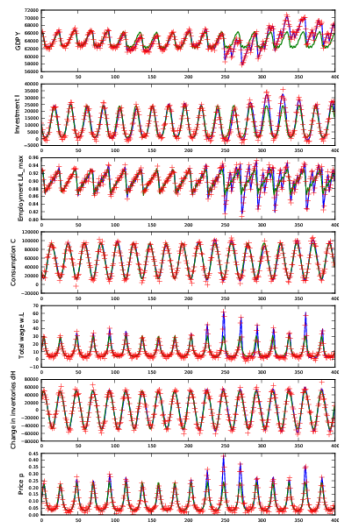
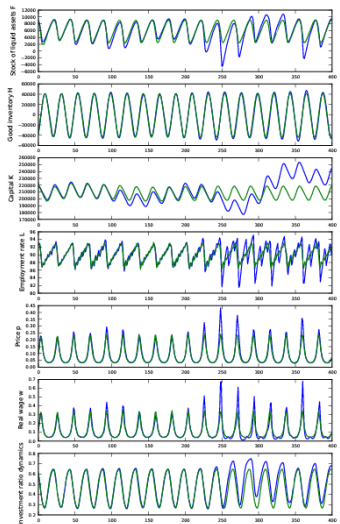
with $M_{k+1 \leftarrow k}$ being NEDyM and η_k is a model error stochastic component which is a fraction of the climatological variability of the prognostic variables.

- ▶ But, as modellers, we are unaware of η_k , but we do know NEDyM:

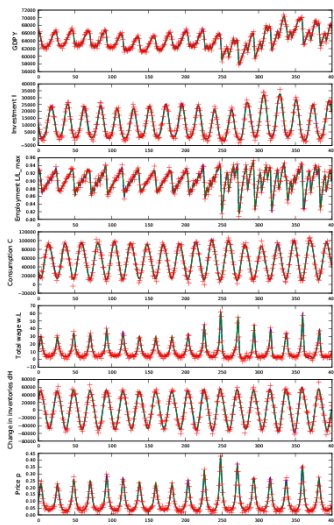
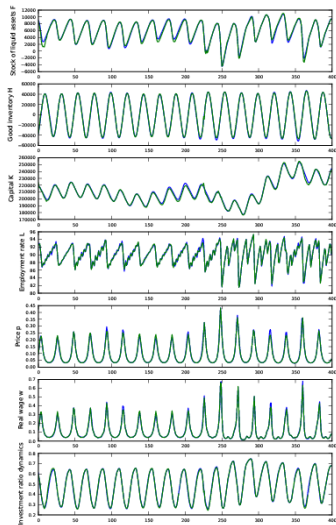
$$\mathbf{x}_{k+1} = M_{k+1 \leftarrow k}(\mathbf{x}_k). \quad (18)$$

- ▶ The underestimation of the model error leads to a divergence of the EnKF, unless one uses an inflation ($\lambda = 1.10$ here).
- ▶ Adaptive inflation filters (EnKF-N, EnKF-ML) are much less prone to diverge.

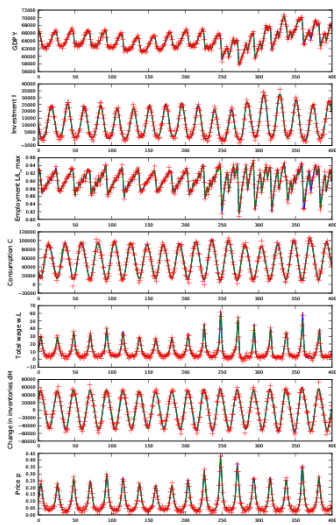
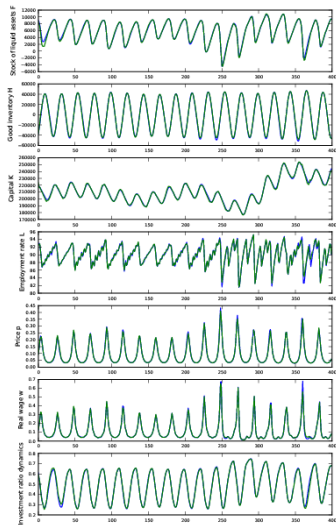
Model error: EnKF (2/4)



Model error: EnKF-N (3/4)

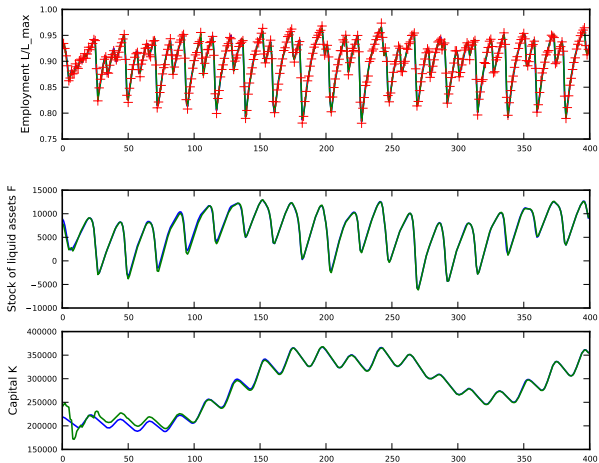


Model error: EnKF with inflation (4/4)



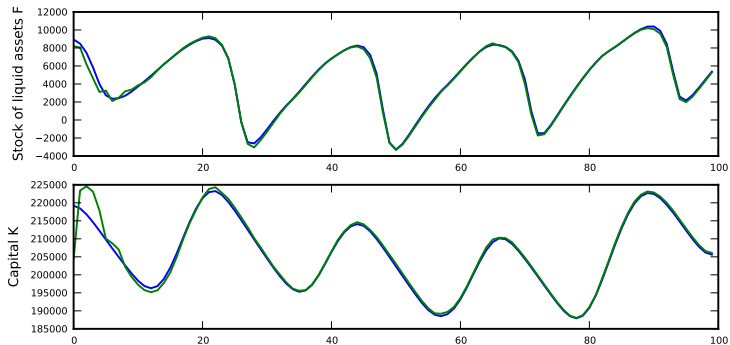
Trackability: observation of a single diagnostic variable (1/2)

- ▶ Observation every quarter of a single diagnostic variable (20% error): the test is failed for all variables **except employment L**
- ▶ EnKF-N almost succeeds with the total wage $w.L$, but for only 7 quarters.



Trackability: other diagnostics (2/2)

- ▶ Observation every quarter of all variables except employment L/L_{\max} and total wage $w.L$: trackable. But fails for a smaller subset.
- ▶ EnKF-N fails in the same condition: disregards too much information.

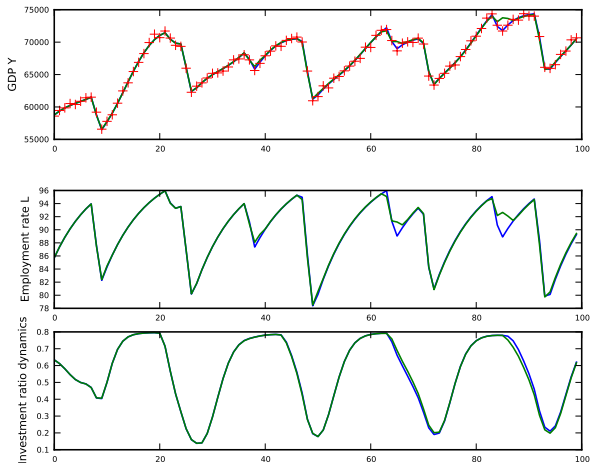


Trackability with more advanced method (1/2)

- ▶ The Iterative ensemble Kalman filter is a lag-one ensemble Kalman smoother (very recent approach).
- ▶ In a nutshell: a semester of data consistently taken into account [Sakov et al., 2012; Bocquet and Sakov, 2012] , instead of one quarter.
- ▶ It does not require the tangent linear and adjoint of NEDyM.
- ▶ It was shown to be very helpful for very nonlinear toy-models.
- ▶ With NEDyM, its usefulness comes from the fact that it is an EnVar method (rather than its efficiently in strongly nonlinear regime).
- ▶ Generalisation to fixed-lag smoother possible [Bocquet, 2012, in preparation], and even more efficient with NEDyM.

Trackability with more advanced method (2/2)

- ▶ Observation every quarter of any variable with IEnKF: the system is now trackable (example of GDP):



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Model prerequisite

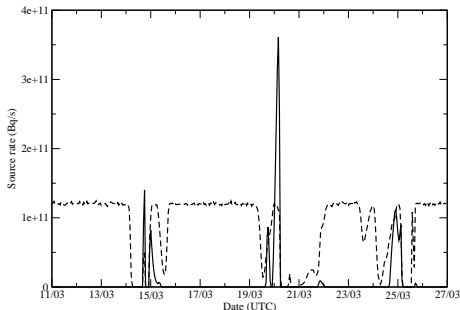
- ▶ The model is stable under very small stochastic perturbations: 1/100 or less of the natural variability.
- ▶ The model is unstable under moderate perturbations.
- ▶ That is insufficient to perform data assimilation with real data.
- ▶ Need to improve the model robustness.

Parameter estimation

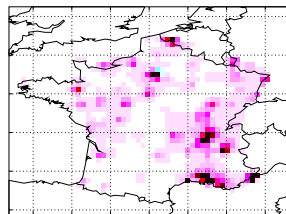
- ▶ Order of magnitude of some of the NEDyM diagnostic variables do not quite match the real economic data. Will a better set of parameter values help?
- ▶ With 13 parameters, it is difficult to obtain a definitive answer by trial and errors.
- ▶ The solution is parameter estimation (through data assimilation).
- ▶ The goal is not only to track and forecast the economy but also estimate the most suited parameters for NEDyM from the observations.
- ▶ A lot of experience in geophysical data assimilation in the field of parameter estimation.

Parameter estimation in geophysics

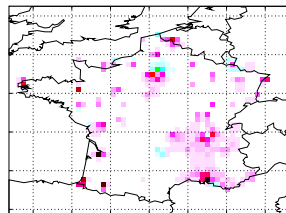
- ▶ Atmospheric chemistry inverse modelling
- ▶ Land surface data assimilation
- ▶ Oceanography
- ▶ Climate, coupled models



Cs-137 Fukushima reconstructed source term [Winiarek et al., 2012]



(a)



(b)

0.50 1.00 2.00

Reconstructed CO emissions [Koohkan and Bocquet, 2012]

Parameter estimation: methods

► Data assimilation methods:

- Kalman filter, extended Kalman filter, extended ensemble Kalman filter and particle filter [Aksoy et al., 2006] [Vossepoel and van Leeuwen, 2007], [Kondrashov et al., 2008], [Wirth and Verron, 2008], [Barbu et al., 2008], [Ruiz et al., 2012] . . . ,
- Simulated annealing, and other stochastic methods [Jackson et al., 2004], [Liu, 2005], [Bocquet, 2012], . . .
- Variational methods [Pulido and Thuburn, 2006], [Bocquet, 2012], . . . ,
- Kalman smoothers.

► Fundamental issue:

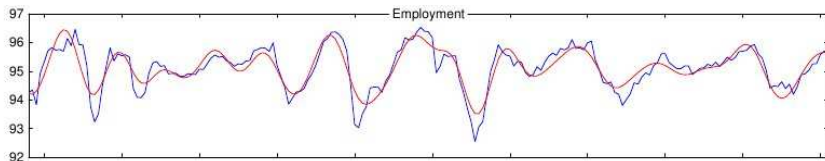
- Do we invert an unobserved parameter or do we just compensate for another source of model error?
- If we compensate, how useful are the results, how do we interpret them?
- For NEDyM: Honestly, we probably don't care about this issue! Maybe later.

Successful assimilation in geophysics: It's all about errors

- ▶ What have we learnt (and still learning) from the use of data assimilation on a wide range of geophysical models?
 - ▶ Our observations are wrong
 - ▶ Our models are wrong
 - ▶ Even when they are so not wrong, they do not tell the same story!
 - ▶ So successful data assimilation is all about errors!
 - ▶ Exception: Global meteorological forecast models are quite good and very well tuned.
- ▶ But severe hardships (modelling: complex microphysics, mathematics: integration of complex microphysics, non-Gaussian statistics) as soon as one has to deal with atmospheric constituents (humidity, gas, aerosols, hydrometeors, ashes), oceanic constituent (ice, salt, algae, plankton, fish, nutrients), or model coupling, models feedback, etc.
- ▶ NEDyM successful data assimilation will require caring for model error. . .

Plans

- ▶ Explore the robustness and flexibility of NEDyM.
- ▶ Twin experiments of parameter estimation for NEDyM
- ▶ Assimilation of real data in NEDyM using an EnKF: very likely to fail.
- ▶ Assimilation of real data in NEDyM using a smoother/variational method.
- ▶ Need for a weak constraint formulation of the data assimilation problem?



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