

# Singular spectrum analysis and synchronization

## *International business cycles*

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*in cooperation with*

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Nogent-sur-Marne

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# Outline

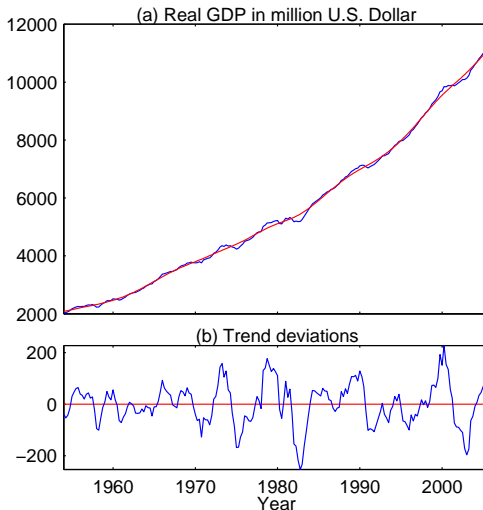
- 1 Macroeconomic activity — “business cycles”
- 2 Singular spectrum analysis (SSA)
- 3 U.S. business cycles
- 4 International business cycles
- 5 Conclusions

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# Macroeconomic activity — “business cycles”

Gross domestic product of the U.S.

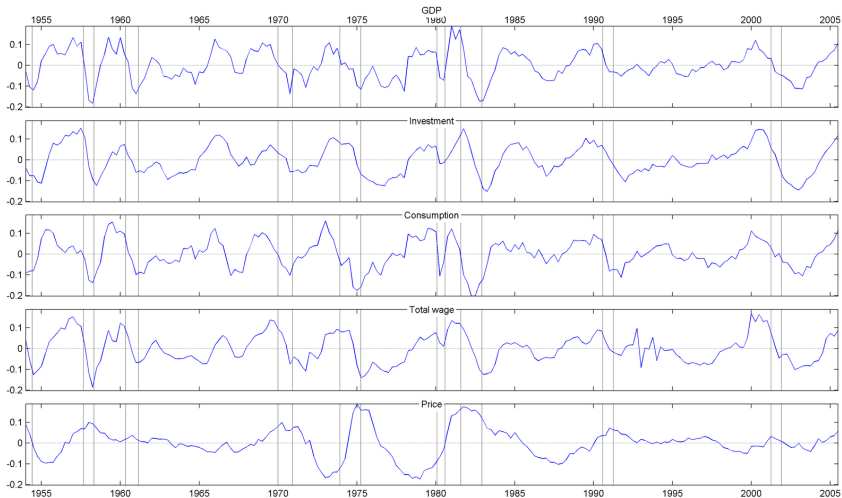


- Around a longterm trend we observe short-terms fluctuations
- Short-term fluctuations are referred to as **business cycles**
- Origin is discussed in controversy:
  - Real business cycle theory (fluctuations arise from exogenous shocks)
  - Endogenous business cycle theory (fluctuations are intrinsic)

U.S. macroeconomic data from the Bureau of Economic Analysis (BEA); detrended with Hodrick-Prescott filter ( $\lambda = 1600$ )

# Macroeconomic activity — “business cycles”

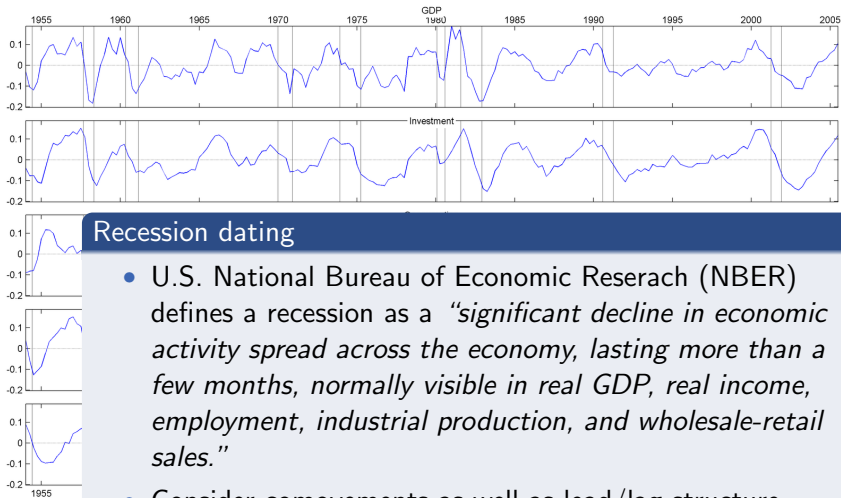
## Macroeconomic indicators of the U.S.



Vertical lines — Recession dates provided by the National Bureau of Economic Research (NBER)

# Macroeconomic activity — “business cycles”

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# Singular spectrum analysis (SSA)

## Concept of SSA

- SSA relies on classical Karhunen-Loève spectral decomposition of a stochastic process as a linear combination of orthogonal functions
- Broomhead and King introduced SSA into dynamical systems analysis [Broomhead and King, 1986a,b]
- Following the Mañé-Takens idea to reconstruct dynamics via time-delayed embedding

**Observation:** Multivariate time series  $\mathbf{x}(n) = \{x_d(n)\}$

$d = 1 \dots D$  channels of length  $n = 1 \dots N$



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**Observation:** Multivariate time series  $\mathbf{x}(n) = \{x_d(n)\}$

$d = 1 \dots D$  channels of length  $n = 1 \dots N$

**Embedding:**  $M$ -dimensional time-delayed embedding of each channel

$$\mathbf{X}_d(n) = (x_d(n), x_d(n+1), \dots, x_d(n+M-1))$$

gives full augmented trajectory matrix

$$\mathbf{X} = \begin{pmatrix} \mathbf{X}_1 & \mathbf{X}_2 & \dots & \mathbf{X}_D \end{pmatrix}$$

# Singular spectrum analysis (SSA)

**Question:** How do we extract important dynamical behavior from

$$\mathbf{X} = \begin{pmatrix} \mathbf{X}_1 & \mathbf{X}_2 & \dots & \mathbf{X}_D \end{pmatrix} ?$$

**Answer:** Dimensionality reduction to principal components

# Singular spectrum analysis (SSA)

## SSA — Principal Component Analysis (PCA) of $\mathbf{X}$

- 1 Compute covariance matrix  $\mathbf{C} = \mathbf{X}'\mathbf{X}/N$
- 2 Eigendecomposition  $\mathbf{\Lambda} = \mathbf{E}'\mathbf{C}\mathbf{E}$ 
  - diagonal matrix  $\mathbf{\Lambda}$  of **eigenvalues**  $\lambda_k$
  - orthogonal matrix  $\mathbf{E}$  of **eigenvectors**  $\mathbf{e}_k$
- 3 Project  $\mathbf{X}$  onto  $\mathbf{E}$  gives **principal components** (PCs)
- 4 Reconstruction of  $\mathbf{x}$  with respect to  $\mathbf{e}_k$  gives **reconstructed components** (RCs)

[Vautard, Yiou, Ghil 1992; Plaut & Vautard 1994; Ghil et al. 2002]

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[Vautard, Yiou, Ghil 1992; Plaut&Vautard 1994; Ghil et al. 2002]

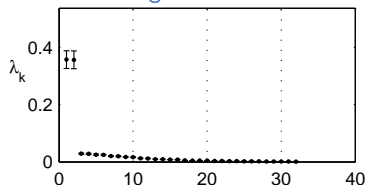
## New to SSA analysis

- Sub-sequent rotation of eigenvectors : reduce mixture effects and improve detection of shared (oscillatory) components

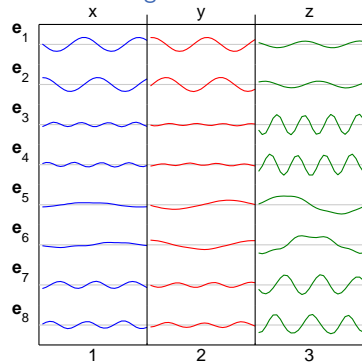
[Groth&Ghil, PRE 2011]

# Reconstruction of oscillatory modes

## Eigenvalues



## Eigenvectors

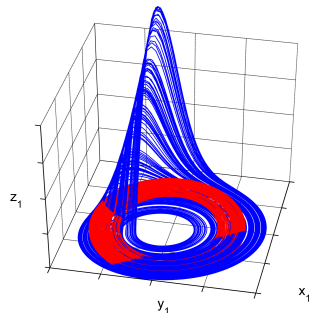


## Eigenvectors

- are data-adaptive, frequency selective filters
- adapt to oscillatory behavior in terms of **oscillatory pairs**

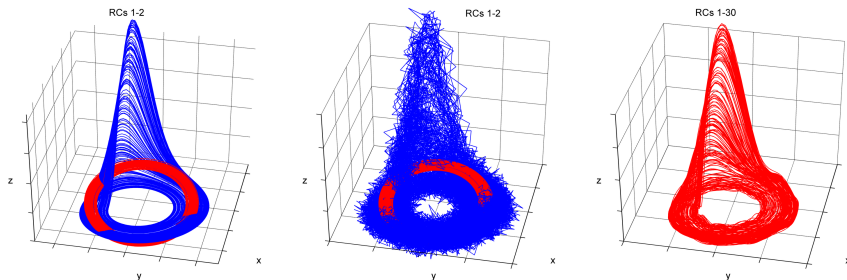
[Vautard&Ghil 1989; Plaut&Vautard 1994]

## Reconstruction with RCs 1–2

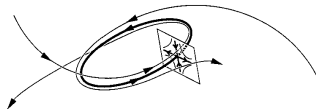


## Separate deterministic behavior from “noise”

- Reconstruct a robust “skeleton” of dynamical system’s structure



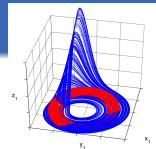
- Ghost limit cycles — albeit unstable, the orbit is visited by system’s trajectory



[Kimoto and Ghil 1993; Ghil and Yiou 1996]

# Cluster synchronization analysis

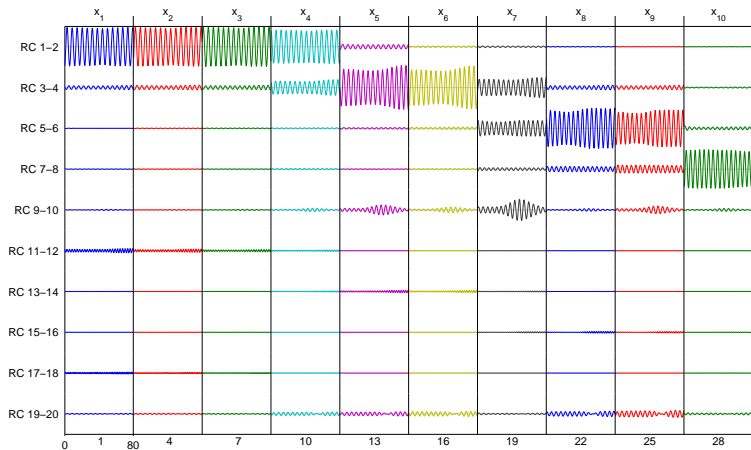
## Chain of chaotic Rössler oscillators



$J = 10$  coupled oscillators; detuned  $\omega_j = 1 + 0.02(j - 1)$



Reconstructed components RCs



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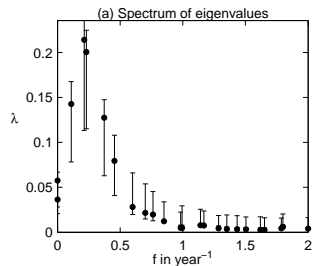
# U.S. business cycles

Deterministic behavior vs. stochastic fluctuations

## Significance test of eigenvalues

- Monte Carlo SSA [Allen& Smith,1996]
- $H_0$  : coupled red noise processes, AR(1); detrended

## SSA analysis of GDP alone



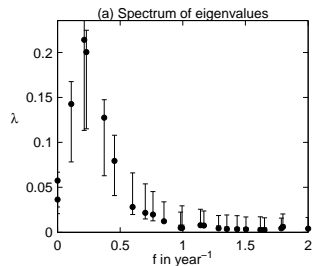
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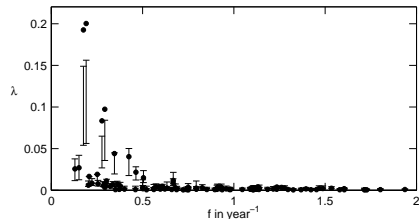
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## SSA analysis of GDP alone



## SSA analysis of 9 aggregates



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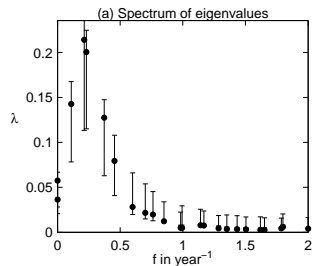
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## Multivariate phenomenon

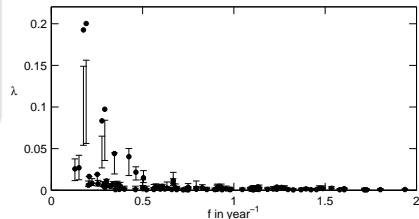
- not limited to GDP variations, but involves all aspects of the economy
- in particular reflected in the comovements

[Groth, Ghil, Hallegatte & Dumas, FEEM working paper 26.2012]

## SSA analysis of GDP alone



## SSA analysis of 9 aggregates

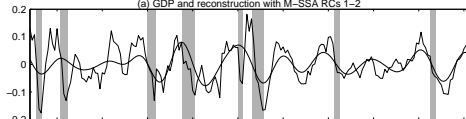


# U.S. business cycles

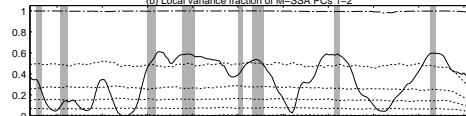
## Deterministic behavior vs. stochastic fluctuations

### Reconstruction

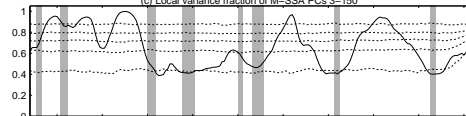
(a) GDP and reconstruction with M-SSA RCs 1-2



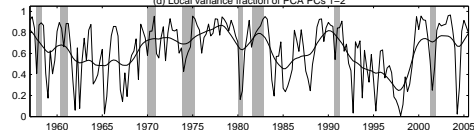
(b) Local variance fraction of M-SSA PCs 1-2



(c) Local variance fraction of M-SSA PCs 3-150



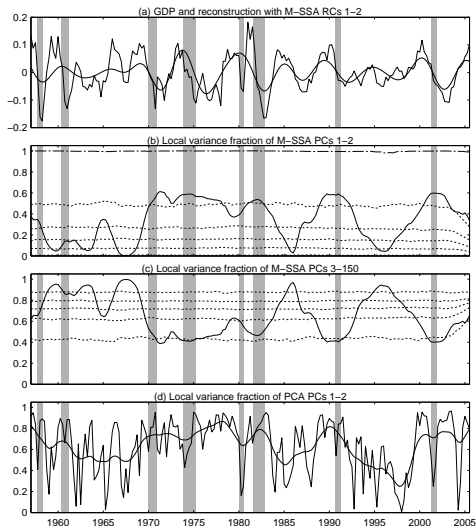
(d) Local variance fraction of PCA PCs 1-2



# U.S. business cycles

## Deterministic behavior vs. stochastic fluctuations

### Reconstruction



### Phase-dependent volatility

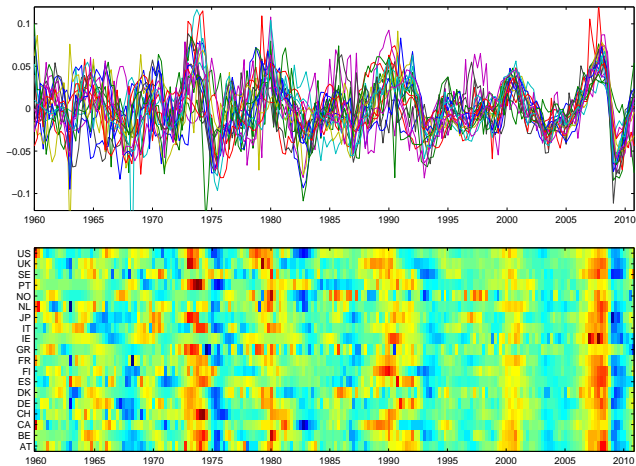
- Recession phase dominated by five-year mode
- Expansion phase exhibits more complex dynamics, with other modes coming into play
- ▶ Vulnerability paradox, NEDyM model
- Groth, Ghil, Hallegatte & Dumas, FEEM working paper 26 (2012)
- Groth, Dumas, Ghil & Hallegatte, *Impacts of natural disasters on a dynamic economy*, submitted

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# International business cycles

Not only a U.S. phenomenon ...

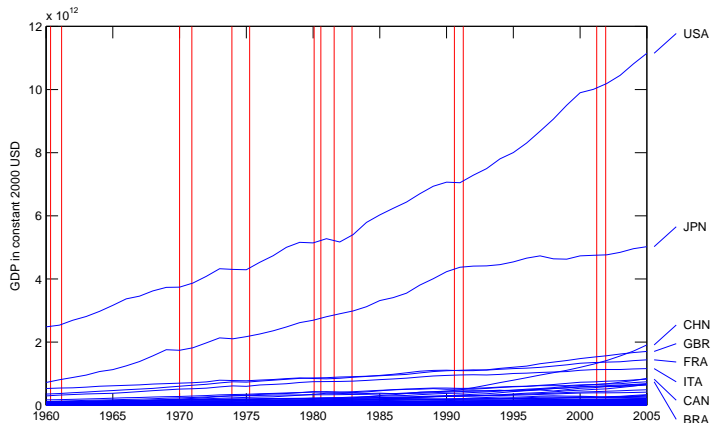


Quarterly GDP from the OECD's national accounts dataset; detrended with Hodrick-Prescott filter ( $\lambda = 1600$ ); standardized

# International business cycles

... but also a worldwide phenomenon

## Annual GDP of 95 countries

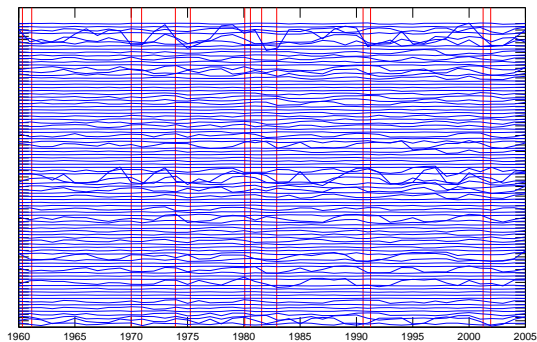


World Development Indicators (WDI) from the World Bank; in constant 2000 US-Dollar



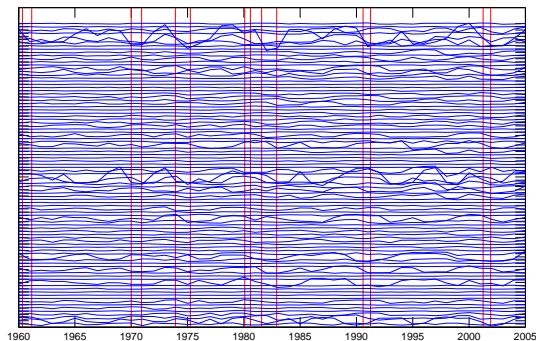
# International business cycles

## Annual GDP of 95 countries — trend residuals

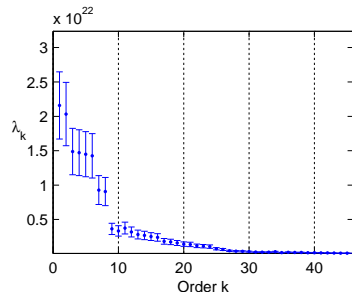


# International business cycles

## Annual GDP of 95 countries — trend residuals



## Eigenvalues

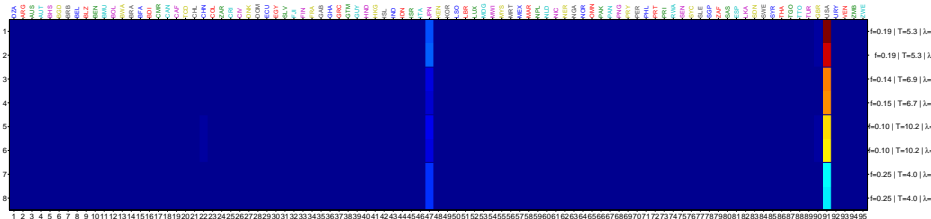


Despite the high complexity of the problem, already 75% of the total variance is explained by the first 8 principal components

## International business cycles

## Cluster participation index of the countries

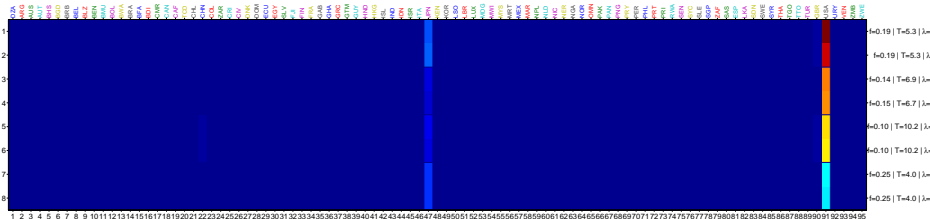
## Absolute index



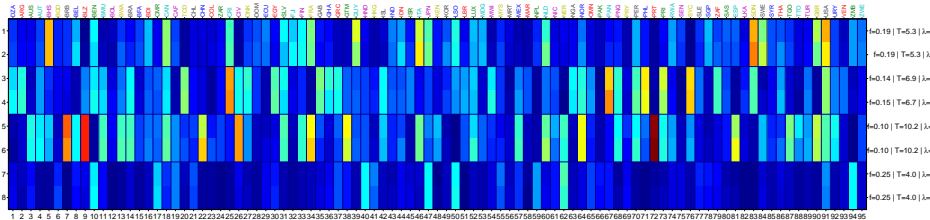
# International business cycles

## Cluster participation index of the countries

### Absolute index



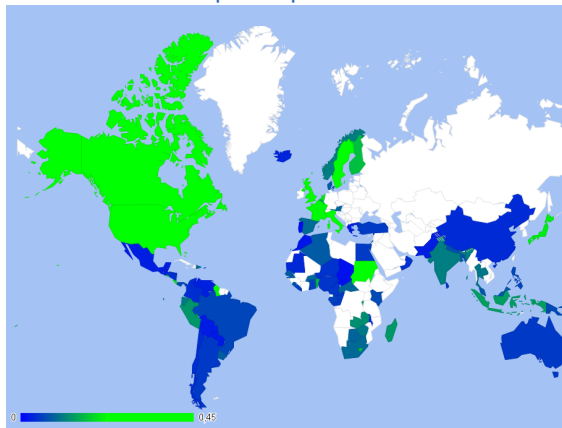
### Relative index



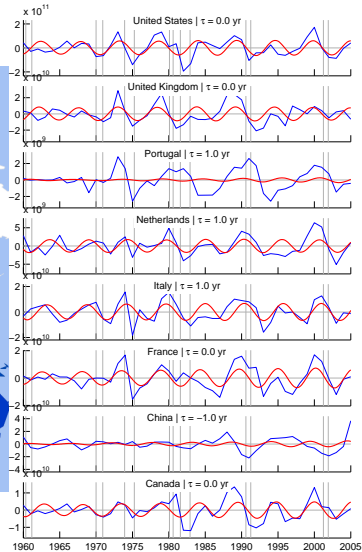
# International business cycles

First five-year mode dominated by the U.S.

## Relative participation index



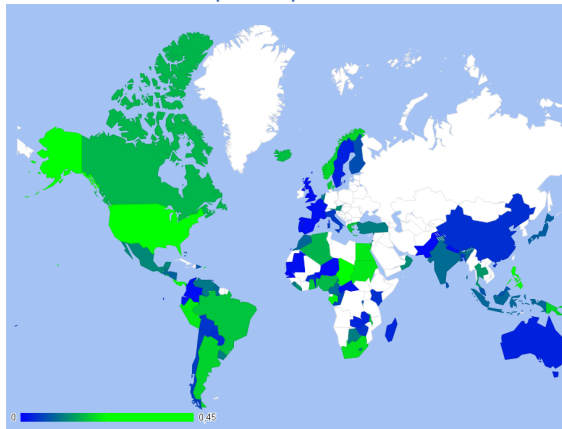
## RCs 1-2



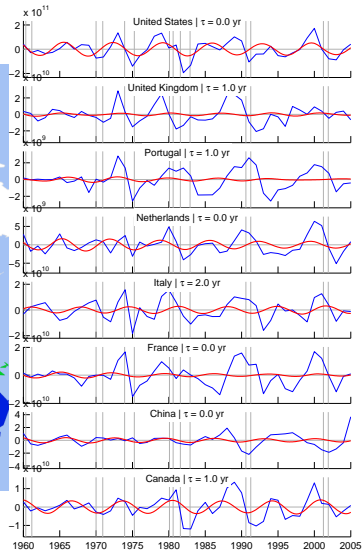
# International business cycles

## Second mode not present in Europe

### Relative participation index



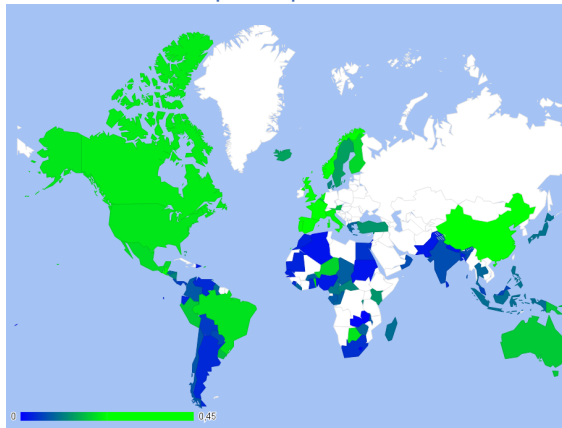
### RCs 3-4



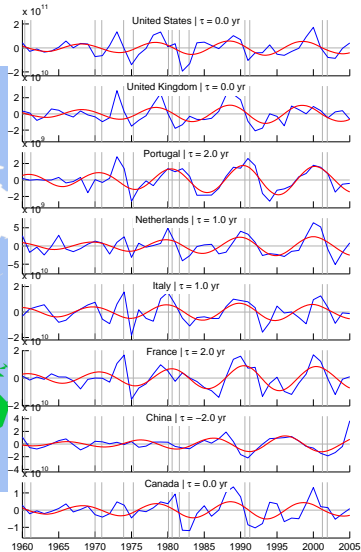
## International business cycles

## Third mode linked to China

## Relative participation index



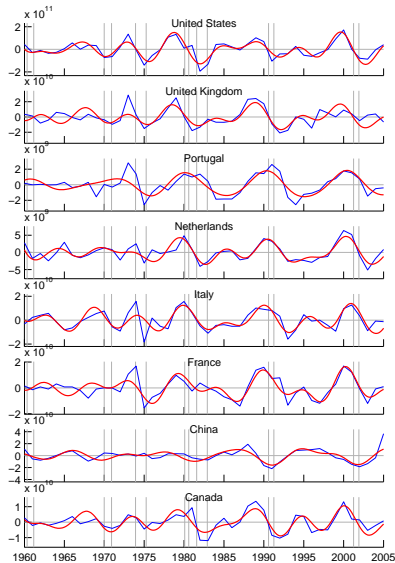
## RCs 5-6



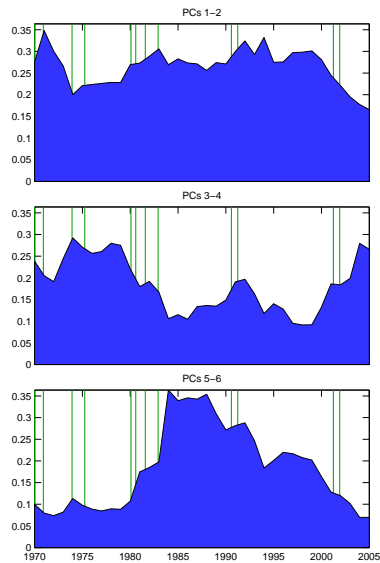
# International business cycles

## Reconstruction with first 3 modes

### RCs 1-6



### Local variance fraction





# Conclusions

## Result

- Presence of genuinely oscillatory — albeit not purely periodic — modes, which are pervasive in many aggregates and countries

## Implication on climate impact assessment and research

- Presence of endogenous business cycle dynamics changes and complicates the response to exogenous shocks and the dynamics of reconstruction
- Overall cost of a natural disaster might depend on the preexisting economic situation
- ▶ These findings raise questions about the assessment of climate change damages or natural disaster losses that are based purely on long-term growth models

## References

- Related work:**
- ▶ Groth & Ghil (2011): "Multivariate singular spectrum analysis and the road to phase synchronisation," *Physical Review E*, 84, 036206
  - ▶ Groth, Ghil, Hallegatte & Dumas (2012): "The role of oscillatory modes in the U.S. business cycle," *FEEM working paper*, 26.2012
  - ▶ Groth, Dumas, Ghil & Hallegatte, "Impacts of natural disasters on a dynamic economy," under review
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  - Broomhead & King (1986b): "On the qualitative analysis of experimental dynamical systems," in *Nonlinear Phenomena and Chaos*, ed. by S. Sarkar, pp. 113–144. Adam Hilger, Bristol, England.
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  - Vautard & Ghil (1989): "Singular spectrum analysis in nonlinear dynamics, with applications to paleoclimatic time series," *Physica D*, 35(3), 395–424.
  - Vautard, Yiou & Ghil (1992): "Singular-spectrum analysis: A toolkit for short, noisy chaotic signals," *Physica D*, 58, 95–126.