TP atelier modelisation, CERES-ERTI

P.Peterlongo

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In TP will explore models in two dimension, i.e. with two dynamic variables. Thus we'll try to draw phase portraits and there will be the possibility to find hopf bifurcation.

We'll work on two new models. One which is a simple model of forest dynamics with fixed budworm density. One which is a prey-predator model which has a hopf bifurcation.

As additional material for this TP, you'll have a brief extract of the book by Strogatz "Nonlinear dynamics and chaos", which you can download from the webpage of the atelier. We'll refer to it as [S].

1. Make sure you are done with work on the budworm model of last TP. You should be able to draw two diagram:

   - An S-shaped bifurcation diagram of B as function of r.
   - A stability diagram such as figure 3.7.5 in [S]. There we see drawn in (r, k) parameter space two curves of saddle node bifurcation points (LP in the language of matcont) which meet in a cusp point (CP).

   In order to review the basic concepts and the model you can read the first seven pages of [S] (pp73-79 of the original). If you have not done yet this diagrams use the 'adimensionalized' form of the model (which is eq (3) in [S] or eq (16) of last TP).

2. Start two work in two dimension. You have three choices of subjects you can work on:

   (a) Work on the forest dynamics model explained in exercise 8.1.10 of [S] (p285 of the original).

   Remarks: you do not need to adimensionalize the model if you do not wish too; to sketch nullclines you can use matlab command line as showed last time; to sketch phase portrait plot some orbits using matcont starting from different initial condition.

   Finally plot also the bifurcation diagram using B as bifurcation parameter.

   (b) Work on prey-predator model of exercise 8.2.8.

   Remarks: Try to work out the questions of the exercise using matlab and matcont. No need to do e) and f), instead plot bifurcation diagram with a as bifurcation parameter (you should find a Hopf bifurcation).

(c) At the end of last TP we suggested to add a slowly evolving dynamic variable (which should represent the slow forest dynamics). The idea was to try to obtain a limit cycle instead of equilibria; the limit cycle should be given by going through the hysteresis cycle.

To do that you should add r as dynamic variable (which maybe does not make a lot of sense from the modelisation point of view) and parametrize its evolution with a linear model.

If we take $y = r/R$ as adimensionalized dynamic variable you should find appropriate constants a, b, c, f with small ω (slow evolution) for the equation:

$$\dot{y} = \omega(ax + by + c) \tag{1}$$

(note that ω is not really needed as a parameter, but it is useful to have freedom in choosing this because $1/\omega$ is the characterstic time of the slow variable y; this explains why it should be small).

Note: it might be good to export to your directory the figure of the diagrams you obtained and note on paper the parameter values and numeric values of bifurcation points you find out (for exporting use menus in figure window, for numeric values use numeric window of matcont while tracing the diagram)